G-SCOP
SCIENCES POUR LA CONCEPTION, L'OPTIMISATION ET LA PRODUCTION

# Star Scheduling

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june 2015

UMR 5272

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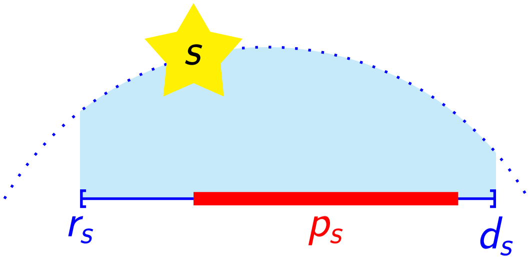
# Looking at the stars

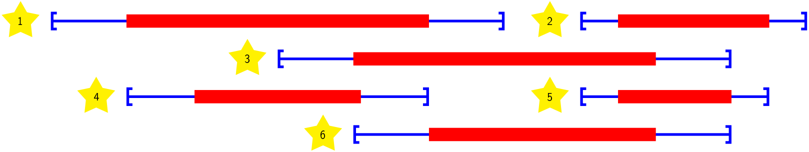



- $[r_s; d_s)$ is the visibility interval
- $p_s$ is the required duration of observation
- $w_s$ is the interest

scheduling the observation of star $s$ means observing $s$ for a continuous duration $p_s$ within the visibility interval $[r_s; d_s)$, rewarding $w_s$

# Star scheduling (one night)

Instance: a set $\mathcal{S}$ of stars; each star $s \in \mathcal{S}$ has an interest $w_s$, an observation duration $p_s$ and a visibility window $[r_s; d_s)$



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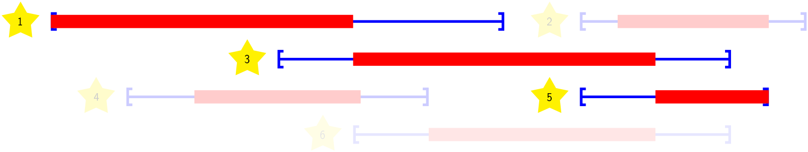
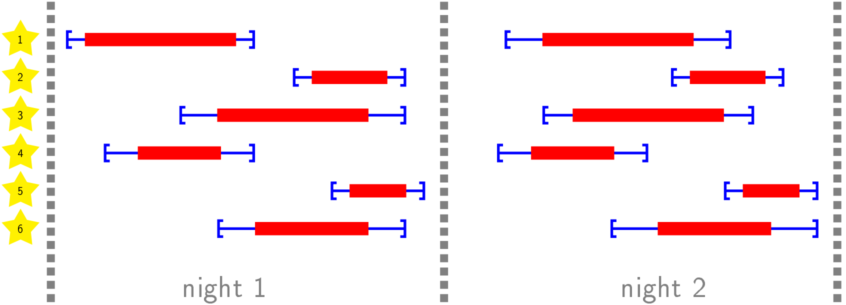



Question: find $\mathcal{S}' \subset \mathcal{S}$ and starting times $t_s, \forall s \in \mathcal{S}'$ such that

- for each $s \in \mathcal{S}'$: $[t_s; t_s + p_s) \subset [r_s; d_s)$
- for each $(s_1, s_2) \in \mathcal{S}'^2$ : $[t_{s_1}; t_{s_1} + p_{s_1}) \cap [t_{s_2}; t_{s_2} + p_{s_2}) = \emptyset$
- $\sum_{s \in \mathcal{S}'} w_s$ is maximized

# Star scheduling (several nights)

Instance: a set $\mathcal{N}$ of nights, a set $\mathcal{S}$ of stars; each star $s \in \mathcal{S}$ has an interest $w_s$, an observation duration $p_s^n$ and a visibility window $[r_s^n; d_s^n)$, depending on the night $n$ of the observation



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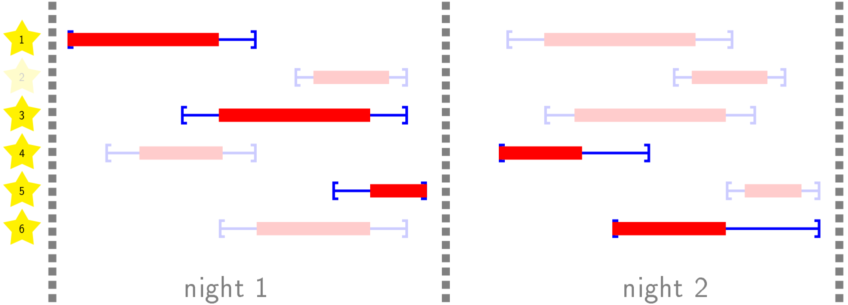
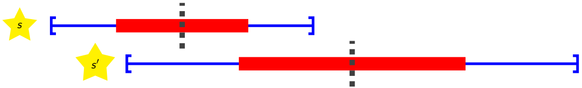


# The order is known!

The meridian instant ($m_s = \frac{r_s+d_s}{2}$) is a mandatory instant of observation, that is: for every star $s$, $p_s^n \geq \frac{d_s^n - r_s^n}{2}$




**Property**

If for all star $s$: $p_s^n \geq \frac{d_s^n - r_s^n}{2}$, then observations must be scheduled by non-decreasing meridian time

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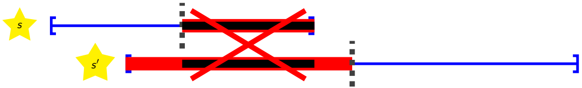


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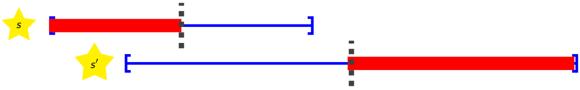


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# A MIP model

$$\max \sum_{s\in\mathcal{S}} w_s \boxed{z_s} \longrightarrow = 1 \text{ iff } s \text{ observed}$$

$$\text{s.c.} \sum_{n\in\mathcal{N}} \boxed{z_s^n} = z_s \longrightarrow = 1 \text{ iff } s \text{ observed on night } n$$

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visibility window of night $n$

$$\left\{ \begin{array}{l} r_s^n z_s^n \leq \boxed{t_s} \longrightarrow \text{starting time of observation } s \\ t_s + p_s^n z_s^n \leq d_s^n z_s^n + M(1 - z_s^n) \end{array} \right.$$

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$s \prec s'$ if observed the same night

$$\begin{cases} z_s^n + z_{s'}^n - 1 \leq \boxed{y_{ss'}} \longrightarrow = 1 \text{ iff } s \text{ and } s' \text{ observed the same night} \\ t_s + p_s^n \leq t_{s'} + M(1 - z_{ss'}) \end{cases}$$

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# Scheduling one night

Instance: a set $\mathcal{S}$ of stars; each star $s \in \mathcal{S}$ has an interest $w_s$, an observation duration $p_s$ and a visibility window $[r_s; d_s)$ such that $p_s \geq (d_s - r_s)/2$; a bound $W$

Question: find a subset of stars so that the total interest is at least $W$, visibility windows are respected and observations do not overlap

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**Complexity of the one night case**

Star scheduling of one night is NP-Hard (even if $w_s = 1$)

# Scheduling of one night is NP-hard

- Variant of Partition : $2n$ pairs $(a_{2i-1}, a_{2i})$ so that $\sum_i a_i = 2B$.
  Can a total of $B$ be made with one item from each pair?

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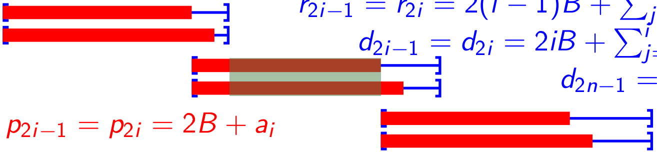
- Set visibility windows so that scheduling a star from a pair does not prevent scheduling a star from another pair and each pair of stars have dedicated instants



$$r_{2i-1} = r_{2i} = 2(i-1)B + \sum_{j=1}^{i-1} \min(a_{2i-1}, a_{2i})$$

$$d_{2i-1} = d_{2i} = 2iB + \sum_{j=1}^{i} \max(a_{2i-1}, a_{2i})$$

$$d_{2n-1} = d_{2n} = (2n+1)B$$

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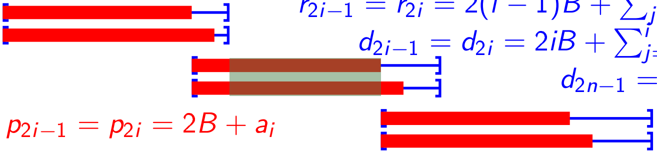
- "yes" to Partition $\iff$ non-idling schedule of length $(2n+1)B$



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# A pseudo-polynomial algorithm

$f(i,t)$: maximum interest with stars 1 to $i$, and such that $s_i$ ends before time $t$

$$f(i,t) = \begin{cases} \min(f(i-1,t), f(i-1,t-p_i)+w_i) & \forall i \in [1,m], t \in [r_i+p_i, T] \\ f(i-1,t) & \forall i \in [1,m], t \in [0, r_i+p_i[ \\ -\infty & \forall i \in [1,m], t < 0 \\ 0 & i = 0, \forall t \in [0,T] \end{cases}$$

We are looking for $f(m,T)$ which can be computed in $O(mT)$

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**Complexity of the several nights case**

Star scheduling of several nights is unary NP-Hard (even if $w_s = 1$ and all nights are identical)

# Logic-based Bender decomposition

Master problem:
assignment of stars to nights

$$\begin{aligned} \max \quad & \sum_{s \in \mathcal{S}} w_s z_s \\ s.c. \quad & \sum_{n \in \mathcal{N}} z_s^n = z_s \end{aligned}$$

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Slave problem:
scheduling of each nights

Night 1: ok?

Night 2: ok?

...

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$$\sum_{s\in C_k^n} z_s^n \leq |C_k^n| - 1$$

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Night 1: ok?

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- efficient MIP
- upper bound at each iteration

- $n$ independent problems
- linear complexity

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# Column generation

Night patterns: $\Omega_n$, set of all possible schedules for night $n$
Pattern $k$ for night $n$: $p^k_{n1} \dots p^k_{n|\mathcal{S}|}$, where $p^k_{ns} = 1$ iff star $s$ belongs to the $k$-th pattern of night $n$; weight $w^k_n = \sum_{s \in \mathcal{S}} w_s p^k_{ns}$

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$$\begin{array}{rcll}
\max \sum_{n\in\mathcal{N}} \sum_{k\in\Omega_n} w^k_n \boxed{\rho^k_n} & \longrightarrow & = 1 \text{ iff pattern } k \text{ used, night } n & \\
\sum_{k\in\Omega_n} \rho^k_n & = & 1 & \forall n \in \mathcal{N} \\
\sum_{n\in\mathcal{N}} \sum_{k\in\Omega_n} p^k_{ns}\rho^k_n & \leq & 1 & \forall s \in \mathcal{S} \\
\rho^k_n \in \{0,1\} & & & \forall n \in \mathcal{N}, \forall k \in \Omega_n
\end{array}$$

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\alpha_n & \sum_{k \in \Omega_n} \rho^k_n & = & 1 & \forall n \in \mathcal{N} \\
\beta_s & \sum_{n \in \mathcal{N}} \sum_{k \in \Omega_n} p^k_{ns} \rho^k_n & \leq & 1 & \forall s \in \mathcal{S} \\
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\end{array}
$$

Reduced cost of $\rho^k_n$: $w^k_n - \alpha_n - \sum_{s \in \mathcal{S}} p^k_{ns} \beta_s$

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\end{array}
$$

Reduced cost of $\rho^k_n$: $w^k_n - \alpha_n - \sum_{s \in \mathcal{S}} p^k_{ns} \beta_s$

Pattern of max reduced cost? Single Night Case with costs $w_s - \beta_s$

# Local search

Classical local-search procedure

Neighborhoods:

- moving a star from one night to another
- inserting an unobserved star
- exchanging two stars

An optimal schedule for each night is computed systematically

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# Solutions

| instance | $\lvert\mathcal{S}\rvert$ | $\lvert\mathcal{N}\rvert$ | BD (OPT/*UB*) val | BD (OPT/*UB*) cpu(s) | CG (UB) val | CG (UB) cpu(s) | LS (LB) val | LS (LB) cpu(s) |
|---|---|---|---|---|---|---|---|---|
| pb1 | 200 | 32 | ***5200*** | 900 | **5200** | 1.47 | **5200** | 0.28 |
| pb2 | 200 | 32 | ***3310*** | 900 | **3310** | 0.99 | **3310** | 0.33 |
| pb3 | 200 | 69 | **7800** | 100 | **7800** | 1.59 | **7800** | 0.17 |
| pb4 | 200 | 69 | - | - | **4870** | 1.63 | **4870** | 0.11 |
| pb5 | 400 | 69 | *12660* | 900 | **11910** | 5.11 | **11910** | 12.39 |
| pb6 | 400 | 69 | *9250* | 900 | 9099.9 | 19.57 | 9070 | 773.95 |
| pb7 | 400 | 142 | - | - | **13680** | 11.85 | **13680** | 0.15 |
| pb8 | 400 | 142 | ***9760*** | 900 | **9760** | 13.71 | **9760** | 0.21 |
| real | 800 | 142 | *18930* | 900 | 18620 | 92.41 | 18510 | 689.60 |
| | | | | | | | 18480 | 306.07 |

# Example solution

solution

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# Star scheduling

- a particular interval-scheduling problem with known order
- NP-hardness is proven for both one night and general cases
- several solution methods are proposed and tested

# Work in progress...

- Instances
  - we need more instances!
  - study the structure of the real instance (e.g., similarity between nights)
- Solution methods
  - enrich numerical experiments
  - embed the CG into a branch & bound to get optimal solutions
  - analyze (and improve) LS behavior
  - get advantage of nights' similarities
- Complexity
  - draw precise complexity frontiers
  - study approximability